20th Australasian Fluid Mechanics Conference
Perth, Australia
5-8 December 2016

Simple Model of Decay of Homogeneous Turbulence Affected by Weak Fluid Acceleration

H. Suzuki[1] **and S. Mochizuki**[1]

[1]Graduate School of Sciences and Technology for Innovation
Yamaguchi University, Yamaguchi, 755-8611 JAPAN

Abstract

This paper discusses the development of a simple model for describing the decay of a homogeneous turbulence subjected to a weak fluid acceleration. A sufficiently weak fluid acceleration may not affect the anisotropy of a decaying homogeneous turbulence. Additionally, the turbulent kinetic energy and its dissipation each follow a power law in homogeneous turbulence when the weak fluid acceleration is absent. Considering these characteristics, the governing equations describing the effect of the weak acceleration on the turbulent kinetic energy and its dissipation are derived. The derived equations are then numerically simulated. When the fluid acceleration is sufficiently small, its effect on the turbulent time scale is negligible. This result simplifies the governing equation for the kinetic energy and yields a simple formula describing the effect of the fluid acceleration. However, the derived simple formula does not necessarily agree with the numerical results. The deviation between the simple formula and the numerical results is considered to be characterized by the decay exponent.

1. Introduction

Decaying axisymmetric homogeneous turbulence is a fundamental type of decaying turbulence that has been observed in the turbulent flows of various engineering problems. The decay of grid-generated turbulence in laboratory experiments is similar to that of axisymmetric homogeneous turbulence. Decaying axisymmetric homogeneous turbulence is also found in homogeneous turbulence distorted by the axisymmetric strain of the mean flow. Turbulence distortion is a classical problem, as discussed by Prandtl [1]. Previous studies [2, 3] have investigated the effects of the mean distortion on decaying homogeneous turbulence. In experiments on grid-generated turbulence, the distortion of the mean flow field has also been used to improve the anisotropy of grid-generated turbulence [4].

Recently, Kurian and Frandsson [5] experimentally studied the effects of weak fluid acceleration on the decay of grid-generated turbulence. The magnitude of the acceleration parameter characterizing the fluid acceleration is small. Although most types of distortion investigated in previous studies have been found to affect the anisotropy of homogeneous turbulence, the distortion caused by weak fluid acceleration, hereafter called the small strain, minimally affects the anisotropy. Decaying nonequilibrium homogeneous turbulence has also been studied [6]. Hurst and Vassilicos experimentally investigated the fundamental characteristics of nonequilibrium homogeneous turbulence [7]. There have been a number of successful studies on nonequilibrium homogeneous turbulence [8, 9, 10, 11, 12]. In the present study, decaying homogeneous turbulence affected by the small strain is considered as a representation of decaying nonequilibrium homogeneous turbulence. Therefore, this paper presents an additional point of view regarding the study of nonequilibrium homogeneous turbulence.

The purpose of this study is to clarify the effects of the small strain on the decay of homogeneous turbulence. Because the strain is small in the present study, RDT (Rapid Distortion Theory) is not appropriate for the present research problem. Therefore, in the present study, the standard k–ε model [13], which is also a classical framework model for turbulence, is applied to solve the present research problem. The application of the k–ε model to solve the present research problem allows the effect of the small strain on the decaying homogeneous turbulence to be described by a mathematical formula. Because the small strain does not affect the anisotropy of the homogeneous turbulence, the governing equations of the k–ε model can be simplified. When the small strain is absent, the turbulent kinetic energy and its dissipation in a decaying homogeneous turbulence each follow a power law. The governing equations describing the effect of the small strain are derived based on this power-law decay.

This study focused on the fact that the small strain negligibly affects the turbulent time scale, as shown in a previous study. When the turbulent time scale is not affected by the small strain, the governing equation that describes the effects can be simplified. By comparing the exact solution of the simplified governing equation with the numerical solution of the original governing equation, this paper discusses the effects of the small strain on the decay of homogeneous turbulence.

2. Methods

The standard k–ε model is applied to determine the effects of the small strain on a decaying homogeneous turbulence. In the decaying homogeneous turbulence affected by the small strain, as shown in a previous study [13], the governing equations of the model take the following simple form:

$$\begin{aligned}\frac{\mathrm{d}k}{\mathrm{d}t'} &= P-\varepsilon \\ \frac{\mathrm{d}\varepsilon}{\mathrm{d}t'} &= C_{\varepsilon 1}P\frac{\varepsilon}{k}-\frac{n+1}{n}\frac{\varepsilon^2}{k},\end{aligned} \tag{1}$$

where k, P, and ε are turbulent kinetic energy, its production term, and its dissipation, respectively, and t' is the dimensional time and $C_{\varepsilon 1} \approx 1.44$ is the model coefficient. When the anisotropy of the homogeneous turbulence is axisymmetric, the production term P in the above governing equations is given as

$$P = P_o\frac{\mathrm{d}U'}{\mathrm{d}x}k(t'),\ P_o = \frac{2(a-1)}{1+2a},\ a = \frac{\langle v^2\rangle}{\langle u^2\rangle} = \frac{\langle w^2\rangle}{\langle u^2\rangle}, \tag{2}$$

where the parameters P_o and a characterize the magnitude of the production term and the anisotropy, respectively. Here u, v, and w are velocity fluctuations for streamwise, transverse, and spanwise directions, and $\langle\ \rangle$ denotes ensemble average.

In decaying homogeneous turbulence that is not affected by the strain of the mean flow, the turbulent kinetic energy and dissipation are described by the following power laws:

$$k(t) = k_o t^{-n}, \varepsilon(t) = \varepsilon_o t^{-(n+1)}, \tag{3}$$

where $t = t'/t'_c$ is the nondimensional time, $t'_c = M/U_o$ is the characteristic time of the bulk flow of a grid-generated turbulence, and n is the decay exponent, where M is the mesh size

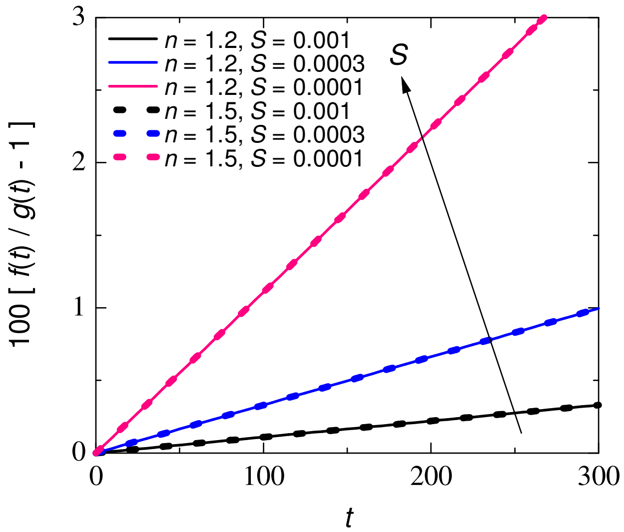

Figure 1: Effect of the small strain on the turbulent time scale. The effect is characterized by $100\,(f(t)/g(t)-1)$. The turbulent time scale increases with increasing small strain. The effect of the small strain on the turbulent time scale is independent of the decay exponent.

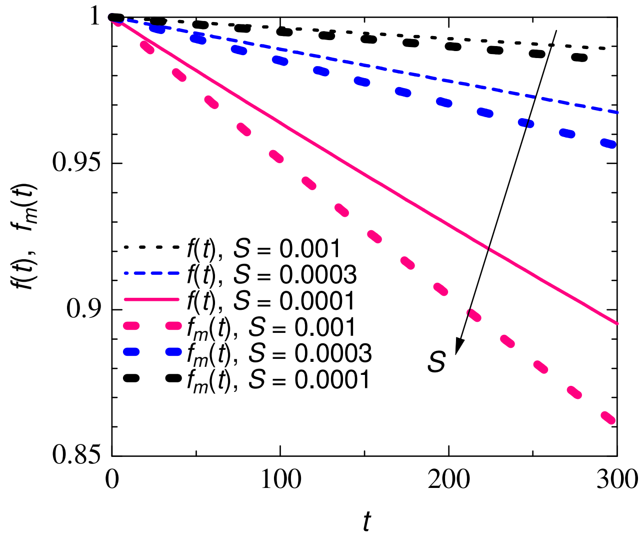

Figure 2: Effects of the small strain on the turbulent kinetic energy f. The small strain of the mean field decreases the turbulent kinetic energy. Temporal variations of $f_m(t)$, which are exact solutions of the simplified equations, are also shown in the figure. The temporal variations of $f_m(t)$ do not agree with those of f.

of a turbulence-generating grid. Additionally, k_o and ε_o are the decay coefficients of the turbulent kinetic energy and its dissipation, respectively, and ε_o is defined as $\varepsilon_o = nU_o k_o/M$. These power laws have been commonly accepted in previous studies. The decay exponent is generally slightly larger than unity and may depend on the Reynolds number of flow field [5].

The turbulent kinetic energy affected by the small strain and its dissipation may not follow these power laws. To characterize the effects of the small strain on these parameters, two nondimensional functions $f(t)$ and $g(t)$ are defined to satisfy

$$k(t) = f(t)k_o t^{-n}, \varepsilon(t) = g(t)\varepsilon_o t^{-(n+1)}. \tag{4}$$

Combining these nondimensional functions with the original governing equations of the model yields

$$\begin{aligned} \frac{\mathrm{d}f(t)}{\mathrm{d}t} &= P_o S f(t) + n\frac{f(t)-g(t)}{t} \\ \frac{\mathrm{d}g(t)}{\mathrm{d}t} &= C_{\varepsilon 1} P_o S g(t) + (n+1)\frac{g(t)}{f(t)}\frac{f(t)-g(t)}{t}, \end{aligned} \tag{5}$$

where $S = (\mathrm{d}U'/\mathrm{d}x)/(M/U_o)$. The effect of the small strain on the turbulent kinetic energy and its dissipation are simulated by solving the above derived governing equations (Eq.(5)). The derived governing equations do not include k_o and ε_o, the magnitudes of which are directly related to the turbulent Reynolds number. Therefore, the solutions of $f(t)$ and $g(t)$ depend on these two quantities.

The derived governing equation is numerically solved using the standard fourth-order Runge–Kutta scheme. The derived equation is incorporated up to $t = 300$. The derived governing equation includes the coefficients and parameters. In the present simulation, the model coefficient $C_{\varepsilon 1}$ is set to 1.44, the decay exponent is set to $n = 1.2$ or 1.5, and the anisotropy is $a = 0.5$, which yields $P_o = -0.5$. The value of the anisotropy is determined based on the experimental results of previous studies in which grid-generated turbulence is measured (e.g., [4]). Following the previous study, S is set to 1×10^{-3}, 3×10^{-4}, and 1×10^{-4}.

3. Results

3.1 Time Scale

First, the effect of the small strain on the turbulent time scale is examined. When the small strain is absent, the following simple equation for the turbulent time scale can be derived:

$$\frac{k(t)}{\varepsilon(t)} = \frac{M}{nU_o}t. \tag{6}$$

Using the two nondimensional functions, the effect of the small strain on the turbulent time scale is given by

$$\frac{k(t)}{\varepsilon(t)} = \frac{f(t)}{g(t)}\frac{M}{nU_o}t, \tag{7}$$

where $f(0) = g(0) = 1$ at the initial state. Therefore, as shown above, the ratio of the two functions describes the effect of the small strain on the time scale.

The effect of the small strain on the time scale is described by the ratio between $f(t)$ and $g(t)$. When S is small, the effect of the small strain on the time scale may be negligible as shown in Figure 1. When the time scale is not affected by the small strain, the following simple relation is true:

$$\frac{f(t)}{g(t)} = 1. \tag{8}$$

$f(t)$ and $g(t)$ are unity in the initial state, and when the effect of the small strain is negligible, $f(t)$ and $g(t)$ remain at their initial values of unity.

The focus of this study then turned to the term $(f(t)-g(t))/t$ in the derived governing equation (Eq. (5)). When the small strain does not affect the time scale, the following condition is true:

$$\frac{f(t)-g(t)}{t} = 0. \tag{9}$$

This condition simplifies the derived equation of $f(t)$ (Eq. (5)) as follows:

$$\frac{\mathrm{d}f(t)}{\mathrm{d}t} = P_o S f(t). \tag{10}$$

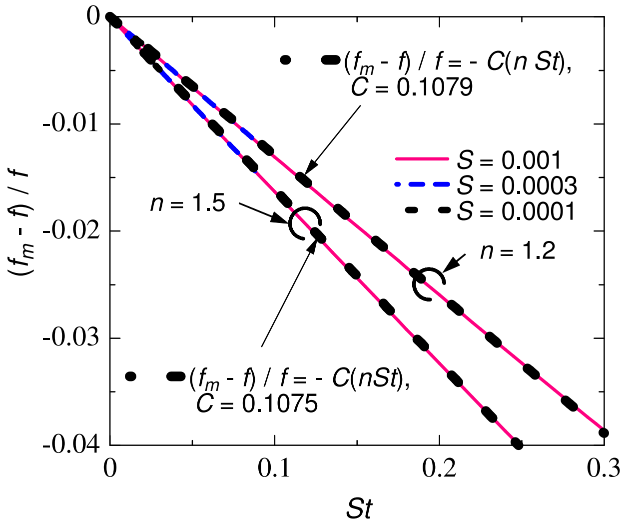

Figure 3: Relative difference $(f_m(t) - f(t))/f(t)$ between $f(t)$ and $f_m(t)$. The temporal variation of $(f_m(t) - f(t))/f(t)$ depends on the decay exponent. The relative differences at different values of S can be approximated by linear functions that are directly characterized by the decay exponent.

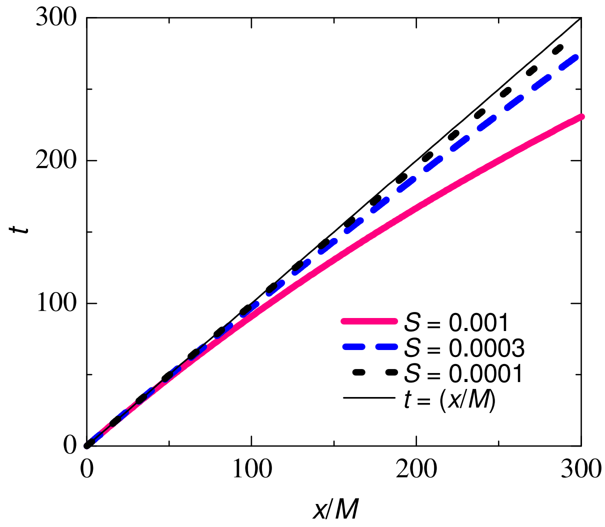

Figure 4: Relationship between the nondimensional time t and the streamwise direction normalized by x/M. This relationship depends on the value of S. The nondimensional time t is smaller than x/M because of the presence of S in Eq. (19).

This equation is a simple ordinary differential equations. Although the original equation of $f(t)$ includes $g(t)$, when the time scale is unaffected by the small strain, the differential equation of $f(t)$ simplifies to a linear equation.

3.2 Exact Solution

The simplified governing equation of $f(t)$ (Eq. (10)) can be solved analytically. The exact solution of $f(t)$ is derived as

$$f(t) = C_o e^{P_o St}, \tag{11}$$

where C_o is a constant of integration. The solution of $f(t)$ (Eq.(11)) is based on an exponential function. A formula for the constant of integration is then obtained. In the present simulation, $f(0)$ is set to unity. This condition yields $C_o = 1$. By applying this condition to the exact solution of $f(t)$, the complete solution $f_m(t)$ is derived as

$$f_m(t) = e^{P_o St}. \tag{12}$$

The simplicity of the solution arises from the fact that the effect of the small strain on the time scale is negligible. When this effect is not small, the formula for $f(t)$ is more complex.

The derived solution $f_m(t)$ is then validated by comparison with numerical results. Figure 2 compares the temporal evolution of $f_m(t)$ with the numerical results of $f(t)$. As shown in Figure 2, $f(t)$ is smaller than unity. This indicates that the small strain of the mean flow reduce the turbulent kinetic energy. The deviation of $f(t)$ from unity increases with increasing S. $f_m(t)$ is also smaller than unity, and its deviation from unity also increased with increasing S. Therefore, the temporal evolution of $f_m(t)$ agrees qualitatively with that of $f(t)$. However, the absolute deviation of $f_m(t)$ from unity is larger than that of $f(t)$.

The disagreement between the temporal evolutions of $f_m(t)$ and $f(t)$ is discussed here. The relative difference between $f_m(t)$ and $f(t)$ is calculated as $(f_m(t) - f(t))/f(t)$. Figure 3 shows the temporal evolution of $(f_m(t) - f(t))/f(t)$ at different values of S. As shown in Figure 3, the temporal evolution of the relative difference collapsed when (St) is used as the independent variable instead of t. The relative difference $(f_m(t) - f(t))/f(t)$ depends on the decay exponent n. The relative difference at $n = 1.5$ is larger than that at $n = 1.2$. Therefore, the absolute value of $(f_m(t) - f(t))/f(t)$ increases with increasing n.

The temporal evolution of $f_m(t)$ is similar to each linear evolution. This is because the magnitude of S is small. Specifically, the following relation is true when S is small:

$$f_m(t) = e^{P_o St} = 1 + P_o St + \cdots. \tag{13}$$

The temporal evolution of $f(t)$ can also be approximated as a linear function, although the exact mathematical formula of $f(t)$ is obtained here. Therefore, the following function is defined:

$$\frac{f_m(t) - f(t)}{f(t)} = -C \times nSt, \tag{14}$$

where C is a constant. Setting this constant to the appropriate value can fit the above linear function (Eq.(14)) to the numerical results with sufficient accuracy, as shown in Figure 3, where the value of C is calculated using least-squares fitting. It should be noted that the value of C negligibly depends on the value of the decay exponent, as shown in Figure 3. This independence suggests that the relative difference between $f(t)$ and $f_m(t)$ is proportional to the decay exponent n.

4. Discussion and Conclusion

4.1 Discussion

The present results are discussed here in relation to a previous study by Kurian and Frandsson [5] (hereafter referred to as KF2009). In the present study, the small strain causes the turbulent kinetic energy to decrease. This result agrees qualitatively with the results of KF2009. As shown in a result of KF2009, the Taylor length scale, which relates the turbulent time scale directly, is negligibly affected by the small strain. The present results suggest that the effect of the small strain on the turbulent time scale is due to the small magnitude of the strain and does not depend on the decay exponent. Based on the results of KF2009, the governing equations of the effect of the small strain on the turbulent kinetic energy indicate that the effect on the time scale is negligible; this is in agreement with the results of the present study. The present study suggests that the governing equation derived by assuming this effect is negligible may not describe the effect on the turbulent kinetic energy. Although the effect of the small strain on the turbulent time scale is small, this small effect must be included to accurately describe the effect on the turbulent kinetic energy.

The relationship between the nondimensional time t and the streamwise direction is also considered. Homogeneous turbulence, which is the type of turbulence considered in this study, decays as a function of time. In contrast, grid-generated turbulence, which is similar to decaying homogeneous turbulence with a constant convection velocity, decays as a function of distance in the streamwise direction [14]. Here, the distance in the streamwise direction is given by $(x/M - x_o/M)$ with respect to the virtual origin x_o/M. Because the velocity in the streamwise direction varies with the distance, the nondimensional time t is not equal to $(x/M - x_o/M)$. Using the normalized streamwise velocity, the nondimensional time is given as

$$t = \frac{x/M - x_o/M}{U/U_o}. \tag{15}$$

Additionally, S can be defined using an ordinary differential equation; therefore, the velocity distribution characterized by S is described as

$$U(x/M) = U_o \left(1 + S\frac{x}{M}\right). \tag{16}$$

Combining Eqs.(15) and (16), the nondimensional time is given by

$$t = \frac{x/M - x_o/M}{1 + S(x/M)}. \tag{17}$$

Figure 4 shows the nondimensional time as a function of x/M As shown in Figure 4, the deviation of Eq.(17) from $t = x/M$, which is obtained when $U = U_o$, is not small.

4.2 Conclusion

This paper discussed the effect of the small strain of the mean flow on the decay of a homogeneous turbulence using the standard k–ε model. First, the governing equations of the effect of the small strain are derived. When the strain is sufficiently small, the effect of the small strain on the turbulent time scale can be negligible. This result is not sensitive to the decay exponent. In this case, the original governing equation describing the effect on the turbulence can be simplified to an equation that can be solved analytically. The exact solution of the simplified equation deviates from the numerical results of the original governing equation. The relative difference between the two solutions, which varies linearly with time, is found to be proportional to the decay exponent. Additionally, a related discussion of the present results is given.

In future work, the strong assumption, which yielded the original governing equation describing the effects of the small strain on the time scale, will be further specified. Furthermore, the results in which the relative difference between the exact solution, which is yielded by modeling the governing equation, and the numerical results is proportional to the decay exponent will be discussed. The coefficient characterizing the relative difference between the linear evolutions will also be determined.

Acknowledgements

The present study is supported in part by the Japanese Ministry of Education, Culture, Sports, Science and Technology through Grants-in-Aid (Nos. 25289030, 25420115, 15K05792, 15K13871, and 15K17970).

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